

Isospin singlet (pn) pairing and quartetting contribution to the binding energy of nuclei

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Isospin singlet (pn) pairing as well as quartetting in nuclei is expected to arise near the symmetry line $N = Z$. Empirical values can be deduced from the nuclear binding energies applying special filters. Within the local density approximation, theoretical estimates for finite nuclei are obtained from results for the condensation energy of asymmetric nuclear matter. It is shown that the isospin singlet condensation energy drops down abruptly for $|N - Z| \approx 4$ for medium nuclei in the region $A = 40$. Furthermore, α -like quartetting and the influence of excitations are discussed.

Keywords: proton-neutron pairing, superfluidity, strongly coupled systems, nuclear matter, α -particle matter.

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I. INTRODUCTION

The structure of nuclei is understood to a large extent within a single-particle approach, the nuclear shell model which was introduced in nuclear theory 50 years ago [1]. Its microscopic foundation should be given from a quantum statistical treatment of the many-nucleon system, where in general also the concept of temperature can be introduced to describe excited nuclei in a dense medium. Within the mean-field approximation, the single-particle approach can be obtained [2].

However, due to the nucleon-nucleon interaction also correlations occur which cannot be accounted for within this quasi-particle approach. Two-particle correlations and higher order correlations as precursors of corresponding bound states may become of importance in regions where the nucleon density is low. A systematic treatment of the effect of correlations can be given in terms of the spectral function, see, e.g., [3,4] and further references given therein.

Free two-nucleon states are described by scattering phase shifts or, more generally, by the T matrix. In the isospin singlet ($S = 1, T = 0$) channel where the interaction is stronger than in the isospin triplet ($S = 0, T = 1$) channel, a bound state, the deuteron, is formed. Two-nucleon states in nuclear matter are strongly modified when increasing the density. In particular, the bound state will disappear at the so-called Mott density [5,6].

Another aspect of the nucleon-nucleon interaction is the formation of quantum condensates. At low temperatures it is well known that in nuclei, nuclear matter, and neutron matter (neutron stars) superfluidity can arise in the isospin triplet channel [2,7,8]. Nucleonic pairing is a well-known effect in the structure of nuclei. In the isospin triplet channel the influence of proton-proton (pp) or neutron-neutron (nn) pairing on the binding energy of nuclei has extensively been investigated, cf. [2].

However, under certain conditions the interaction in the proton-neutron (pn) isospin singlet channel may be even stronger. For instance, in the two-particle system a bound state, the deuteron, arises. In nuclear matter, at densities below the Mott density a bound state (quasi-deuteron) can exist so that at low temperature the Bose-Einstein condensation of these quasi-deuterons may occur. An interesting feature of the isospin singlet pairing in symmetric nuclear matter is the cross-over from Bose-Einstein condensation of deuterons at low densities to BCS neutron-proton pairing at high densities [9,10].

In spite of the relatively strong interaction, triplet pairing seems less apparent in nuclear structure systematics (see the studies of Goodman on $T = 0$ pairing in nuclei [11]). However, it should be essential for nuclei near the symmetry line $N \approx Z$. This phenomenon becomes of importance for heavier $N \approx Z$ nuclei [12] as will be produced in the new radioactive beam facilities.

A further interesting effect is the possible occurrence of higher order condensates such as α -like quartetting [13,14]. At present, the signatures of isospin singlet pairing and the relation to quartetting have not so clearly been worked out. An interesting point is to identify signatures of quartetting in finite nuclei, see [15]. One of the possible consequences

is the contribution of isospin singlet pairing and four-particle correlations to the binding energy of nuclei, that will be discussed below.

The treatment of correlations in the many-nucleon system is more simple for infinite nuclear matter. The new results with respect to isospin singlet pairing and quartetting [15] should be of interest for the evaluation of binding energies of finite nuclei as well. One possible way to apply results of infinite matter calculations to finite systems is the local density approximation (LDA) which was elaborated successfully in atomic, molecular and condensed matter systems, see [16] and references therein quoted. We will use this approach to perform exploratory calculations in finite nuclei. Compared with shell-model calculations, the LDA is by far more simple and has been proved to give adequate results in the case of pp and nn pairing [17]. We will use this approach to give estimates for the effects of isospin singlet pairing and quartetting on nuclear binding energies.

The question of different kinds of pairing in proton-rich nuclei has been investigated by various authors. In addition to the $T = 1$ pairing, $T = 0$ np -pairing modes have been investigated to explain the Wigner energy in $N = Z$ nuclei [18]. A more extended discussion of the origin of the Wigner term is given in [19] where a method has been developed to extract the Wigner term from experimental data. Both, empirical arguments and shell-model calculations suggest that the Wigner term can be traced back to the isospin $T = 0$ part of nuclear interaction. However, it has also been found that the Wigner term cannot be solely explained in terms of correlations between the neutron-proton $J = 1, T = 0$ (deuteron-like) pairs. One should, however, realize that in a finite nucleus the intrinsic quantum numbers of the deuteron can be mixed with the orbital motion of the nucleons in the shell model potential. Recently, pairing and the structure of the pf -shell $N \approx Z$ nuclei have been discussed in [20]. There, the isovector and isoscalar pairing interaction has been studied. It has been found that the Wigner energy cannot purely be explained as a pairing effect, considering only the zero angular momentum ($L = 0$) channel.

Calculations of the pairing and isospin symmetry in proton-rich nuclei were performed [21]. Near $N \approx Z$, a steep decrease of the isoscalar proton-neutron pairing energy is found with increasing $|N - Z|$. An interesting fact is that in ^{50}Cr proton-proton and neutron-neutron pairing is more reduced with temperature than the isoscalar proton-neutron pairing [22].

Proton-neutron pairing has been discussed also with respect to other properties, see [15]. For instance, a cranked shell model for the description of rotational bands in $N \approx Z$ nuclei has been formulated in [23] taking the isovector proton-neutron pairing explicitly into account. More detailed discussions with respect to the binding energies can be found in [14]. Proton-neutron pairing and its relation to isospin symmetry has been studied within the BCS theory in [24]. Recently, the role of the proton-neutron interaction in $N \approx Z$ nuclei and its consequences for pn -pairing has been investigated in [25].

We will proceed as follows: In Section 2 we analyze the binding energies of nuclei near $N = Z$. The basic formalism of LDA is given in Section 3. We proceed then with the presentation of numerical results in Section 4 and draw some conclusions in Section 5.

II. BINDING ENERGIES OF NUCLEI NEAR $N = Z$

A. Filters for isospin singlet condensates

The empirical binding energies of nuclei are well approximated by the Bethe-Weizsäcker formula, containing the contributions of bulk, surface, Coulomb and asymmetry energy. Additional contributions originate from pairing in the isospin triplet channel (pp , nn) and shell structure effects, for instance the behavior near magic numbers.

We assume that the binding energy $B(Z, N)$ can be decomposed into different contributions as

$$B(Z, N) = B_{\text{bulk}}(Z, N) + B_{\text{surf}}(Z, N) + B_{\text{Coul}}(Z, N) + B_{\text{asymm}}(Z, N) + B_{\text{shell}}(Z, N) + B_{\text{pair}}(Z, N) + \Delta B(Z, N) . \quad (1)$$

The term $\Delta B(Z, N)$ contains the effects of correlations between protons and neutrons (isospin singlet (pn) pairing, quartetting) which are of interest here. In nuclear matter, the occurrence of different condensates has been intensively investigated. Isospin singlet (pn) pairing is strong in symmetric nuclear matter for densities up to the saturation density. The dependence of the pn pairing gap on the asymmetry of nuclear matter for different densities has been investigated in [26,27]. If the densities of protons and neutrons, or the corresponding chemical potentials μ_p , μ_n are sufficiently different, a condensate cannot be formed.

Analogously, the effects of isospin singlet (pn) pairing and quartetting are expected to be strong near $N = Z$, and disappear if the absolute value of the difference $N - Z$ becomes large. Therefore, it seems to be reasonable to investigate the behavior of

$$\Delta B(Z, N) = b_{N-Z}(Z) \quad (2)$$

in dependence of $N - Z$, considering Z as a parameter.

To select out the contribution $\Delta B(Z, N)$ to the binding energy, filters can be applied which eliminate the other contributions given in (1) to a large extent. We consider the following (horizontal and vertical) filters

$$\begin{aligned} h(Z, N) = & 2B(Z, N) - B(Z, N - 2) - B(Z, N + 2) \\ & - 2B(Z - 2, N) + B(Z - 2, N - 2) + B(Z - 2, N + 2), \end{aligned} \quad (3)$$

$$\begin{aligned} v(Z, N) = & 2B(Z, N) - B(Z - 2, N) - B(Z + 2, N) \\ & - 2B(Z, N + 2) + B(Z - 2, N + 2) + B(Z + 2, N + 2). \end{aligned} \quad (4)$$

These simple filters contain the difference of second order differences. Therefore, smooth dependences on Z, N nearly cancel out, up to second order. This concerns not only the contributions of bulk, surface, and Coulomb energy, but also the asymmetry term. The isospin triplet pairing effects compensate because only nuclei of equal parity are considered. Similarly, shell effects compensate to a certain extent assuming that they are related only to the respective values of the numbers Z or N .

From these primitive filters (3), (4), other filters can be constructed by superposition. The filter used in [18] to evaluate the Wigner energy can be constructed as

$$W(A) = -\frac{1}{8} v\left(\frac{A}{2}, \frac{A}{2} - 2\right) - \frac{1}{8} h\left(\frac{A}{2}, \frac{A}{2} - 2\right), \quad (5)$$

or, more generally

$$w(Z, N) = -\frac{1}{8} v(Z, N - 2) - \frac{1}{8} h(Z, N - 2). \quad (6)$$

Another, more symmetric filter

$$g(Z, N) = \frac{1}{8} h(Z, N) - \frac{1}{8} h(Z + 2, N) \quad (7)$$

was considered in [28].

We give the results for the filter $h(Z, N)$ in Fig. 1. The dependence of $h(Z, N)$ on Z is shown for different parameter values $N - Z$, where even and odd values of Z are shown separately. It is clearly seen that the filter $h(Z, N)$ becomes small for large $N - Z$. Furthermore, the average value decreases with increasing Z . Also, even-even and odd-odd numbers of Z, N show a different behavior. Similar results can be obtained using the filters (4), (6), or (7).

The relation to the quantities $b_i(Z)$ introduced above can immediately be given. For the filter $h(Z, N)$ we find

$$\begin{aligned} h(Z, N) = & -b_{N-Z-2}(Z) + 2b_{N-Z}(Z) + b_{N-Z}(Z - 2) \\ & - b_{N-Z+2}(Z) - 2b_{N-Z+2}(Z - 2) + b_{N-Z+4}(Z - 2). \end{aligned} \quad (8)$$

Since only differences of the quantities $b_i(Z)$ are determined by the filter $h(Z, N)$, their values can be reconstructed using additional assumptions. To simplify the extraction of the quantities $b_i(Z)$ we assume that $b_{Z-N}(Z)$ is strongly depending on the asymmetry ($Z - N$), but only weakly depending on the mass number (Z), after eliminating a possible even-odd staggering. Therefore, we approximate $b_i(Z - 2) \approx b_i(Z)$ so that $h(Z, N) \approx -b_{N-Z-2}(Z) + 3b_{N-Z}(Z) - 3b_{N-Z+2}(Z) + b_{N-Z+4}(Z)$. According to the increase of asymmetry discussed above, we assume that the $b_i(Z)$ disappear for large absolute values of i . Taking $b_i = 0$ for $|i| \geq 6$, the results for the quantities $b_i(Z)$ are shown in Fig. 2.

A first important result is that the quantities $b_i(Z)$ depend mainly on the absolute value of i . As a consequence, $h(Z, Z - 2)$ nearly coincides with $-h(Z, Z)$, whereas $h(Z, Z - 1)$ is close to zero, in contrast to $h(Z, Z + 1)$. In detail, the values for b_0, b_2, b_4 were found from the solution of equations containing $h(Z, Z)$, $h(Z, Z + 2)$, $h(Z, Z + 4)$, whereas the values for b_1, b_3, b_5 were found from the solution of equations containing $h(Z, Z + 1)$, $h(Z, Z + 3)$, $h(Z, Z + 5)$. To derive the average behavior with respect to Z , we considered the ratios b_i/b_0 as shown in Fig. 3. The average of these values over the whole range of experimentally accessible data ($8 \leq Z \leq 30$) are given in Fig. 4. They show a decrease with increasing $|i| = |N - Z|$, so that b_5 is nearly zero.

B. Further filters

A further interesting property of the quantities $b_i(Z)$ is their dependence on Z . First we are interested in a possible even-odd staggering. This can be deduced from the values shown in Fig. 2. Alternatively, we also can consider, as a special indicator to extract even-odd staggering, the filter

$$c(Z, N) = (-1)^Z [B(Z, N) - B(Z, N+2) - B(Z+2, N) + B(Z+2, N+2) \\ + B(Z+1, N-1) - B(Z+1, N+1) - B(Z-1, N-1) + B(Z-1, N+1)] . \quad (9)$$

We decompose

$$b_i(Z) = \bar{b}_i(Z) + (-1)^Z \delta b_i(Z) \quad (10)$$

and assume, that the remaining dependence on Z is smooth, as already used above. Then we have

$$c(Z, N) = -2\delta b_{N-Z-2}(Z) + 4\delta b_{N-Z}(Z) - 2\delta b_{N-Z+2}(Z) . \quad (11)$$

Now, we can perform the reconstruction of $\delta b_i(Z)$ along the lines given for $b_i(Z)$. The filter $c(Z, N)$ is shown in Fig. 5, the parameters $\delta b_i(z)$ in Fig. 6 as a function of Z , and the averaged values are shown in Fig. 7 vs the asymmetry $i = N - Z$. We see that also there the staggering contribution disappears with increasing $|i|$, however, the statistical errors are large. The global dependence of $\bar{b}_i(Z)$, $\delta b_i(Z)$ on Z cannot easily be extracted. After a steep decrease for small Z a flattening is observed at higher values of Z , see also Figs. 2, 6.

From other approaches, see [19], the Wigner energy E_W is introduced which occurs in the additional binding due to the np -pair correlations

$$B_{np, \text{pair}} = -\epsilon_{np}(A)\pi_{np} + E_W \quad (12)$$

with $\pi_{np} = (1 - (-1)^N)(1 - (-1)^Z)/4$. The first contribution to the np pairing energy (ϵ_{np}) represents additional binding due to the residual interaction between the two odd nucleons in an odd-odd nucleus. The Wigner energy E_W is believed to represent the energy of collective np -pairing correlations. It can be decomposed into two parts:

$$E_W = W(A)|N - Z| + d(A)\pi_{np}\delta_{NZ} \quad (13)$$

Filter used in [18] can be constructed as given by (5) and

$$d(A) = \frac{1}{2} c\left(\frac{A}{2} - 3, \frac{A}{2} - 1\right) + \frac{1}{2} c\left(\frac{A}{2} - 2, \frac{A}{2}\right) . \quad (14)$$

The d -term represents a correction for $N = Z$ odd-odd nuclei. Estimates suggest that the ratio d/W is constant, values 1 and 0.56 have been reported [18]. Both filters can be expressed by superposition in terms of the more simple filters $h(Z, N)$, $v(Z, N)$, $c(Z, N)$ given here. A more general filter is introduced as $d(Z, N) = c(Z-3, N-1)/2 + c(Z-2, N)$ as an average over neighboring values of c . Some values are shown in Fig. 8.

The results of our phenomenological treatment can be summarized as follows (some preliminary results have been presented in [29]):

- (i) There is a contribution $b_i(Z)$ to the nucleon binding energy which is due to proton-neutron correlations of the type of a isospin-singlet pairing or a quartetting condensate.
- (ii) This contribution depends on the absolute value of the asymmetry parameter $|N - Z|$. It has a maximum magnitude for symmetric nuclei $N = Z$ and decreases with increasing asymmetry, disappearing near $|N - Z| = 4$.
- (iii) It shows an even-odd staggering as function of Z .
- (iv) On the average, it decreases with increasing Z , steep for small values of Z , but flat for large Z .

III. LOCAL DENSITY APPROXIMATION

A theoretical interpretation of the contribution to the binding energy due to pn pairing and quartetting could be given by the Local Density Approximation (LDA). In contrast to shell-model calculations, the LDA is by far more simple and has been proved to give adequate results in the case of pp and nn pairing [17].

As well known from quantum statistics of the inhomogeneous fermion gas, the energy and wave function of the ground state of a many-fermion system can be calculated within a variational approach. The energy density is considered as a functional of the fermion density, which, in the case of a nucleonic system, depends in addition to space coordinates also on spin and isospin. It can be decomposed into kinetic, potential, exchange and correlation energy. Within a gradient expansion, in lowest order the energy density depends only on the local values of the nucleon density. As a consequence, the exchange and correlation energy can be approximated using results from nuclear matter calculations.

In particular, the contribution to the exchange and correlation part of the energy due to the formation of a condensate can be evaluated within nuclear matter theory. There is an extended literature on isospin triplet (pp , nn) pairing. More recently, also isospin singlet (pn) pairing has been considered [9], which for symmetric nuclear matter may become stronger compared with isospin triplet pairing at subnuclear densities because of the more attractive nucleon-nucleon interaction in the isospin singlet channel.

A. Pairing vs. quartetting in symmetric matter

A standard way to describe quantum condensates in many-body systems is the method of thermodynamic Green functions. Treating the two-particle Green function in ladder Hartree-Fock approximation, an effective wave equation (in matrix notation) $\psi_\lambda = K_2(E_\lambda) \psi_\lambda$ for the quantum state λ can be derived. Explicitly this reads

$$\psi_\lambda(12) = \sum_{1'2'} K_2(12, 1'2', \epsilon_\lambda) \psi_\lambda(1'2') \quad (15)$$

with

$$K_2(12, 1'2', z) = V(12, 1'2') \frac{1 - f(1) - f(2)}{z - \epsilon(1) - \epsilon(2)}. \quad (16)$$

The influence of the medium is contained in the single-particle energy

$$\epsilon(1) = p_1^2/2m + \sum_2 V(12, 12)f(2) \quad (17)$$

and in the Pauli blocking term $[1 - f(1) - f(2)]$. Here, $f(1) = [\exp\{\epsilon(1)/T - \mu/T\} + 1]^{-1}$ is the Fermi distribution function and '1' denotes momentum, spin, and isospin coordinates, whereas $V(12, 1'2')$ is the antisymmetrized matrix element of the two-body interaction.

The transition to a superfluid state is obtained from the Thouless criterion as described by the Gorkov equation $\psi_2 = K_2(\mu_1 + \mu_2) \psi_2$. Depending on the respective channels considered, it allows the determination of the critical temperatures T_s^c or T_t^c for the isospin singlet and triplet channels, respectively, as a function of the chemical potential.

The solution of the Gorkov equation has been considered by different authors using realistic bare nucleon-nucleon interactions. It has been found that in comparison with the isospin triplet channel, in the isospin singlet channel the transition to superfluidity should arise at relatively high temperatures [9,26,27], see also Figs. 9, 10. This is a consequence of the stronger interaction in the isospin singlet channel which leads to the formation of the deuteron in the low-density limit where $f \ll 1$. Estimates give a value of the critical temperature up to $T_s^c \approx 5$ MeV at one third of the nuclear matter density. At the same time, at zero temperature a large gap arises [9].

In a recent letter [15] it has been shown that in a certain region of density, pairing has to compete with quartetting. It has been found that under certain conditions in symmetric nuclear matter the transition to isospin singlet pairing, which is stronger than triplet pairing, will not occur because the quartetting transition occurs before that. Within a cluster-mean field approach [5,30], the critical temperature for the quartetting transition was obtained from the equation

$$G_4(1234, 1'2'3'4', z) = \frac{f(1)f(2)f(3)f(4)}{g_4(1234)} \frac{\delta_{11'}\delta_{22'}\delta_{33'}\delta_{44'}}{z - \epsilon_4(1234)} + \sum_{1''2''3''4''} K_4(1234, 1''2''3''4'', z) G_4(1''2''3''4'', 1'2'3'4', z), \quad (18)$$

$$K_4(1234, 1'2'3'4', z) = V(12, 1'2') \frac{f(1)f(2)}{g_2(12)} \frac{\delta_{33'}\delta_{44'}}{z - \epsilon_4(1234)} + \text{perm.}, \quad (19)$$

where we use the abbreviation $\epsilon_n(12\dots n) = \epsilon(1) + \epsilon(2) + \dots + \epsilon(n)$, and $g_n(12\dots n) = [\exp(\epsilon_n(12\dots n) - n\mu)/T - 1]^{-1}$ being the Bose distribution function. The instantaneous part of interaction kernel is obtained by using the technique of Matsubara Green functions as where the terms obtained by renumbering are not given explicitly. We have used the identity $\bar{f}(1)\bar{f}(2)\dots\bar{f}(n) - f(1)f(2)\dots f(n) = g_n^{-1}(12\dots n)f(1)f(2)\dots f(n)$ with $\bar{f} = 1 - f$. The solution of the equation $\psi_4 = K_4(4\mu)\psi_4$ gives the critical temperature for the onset of quartetting as a function of the chemical potential as shown in Fig. 9, or the density as shown in Fig. 10. Within an estimate by using a variational calculation, the transition to quartetting beats the transition to isospin singlet pairing if the density is smaller than 0.03 fm^{-3} , see Fig. 10.

B. Gap equation and condensation energy for asymmetric nuclear matter

For infinite nuclear matter, the gap energy at zero temperature as well as at finite temperature has been investigated for pairing in the different channels in dependence on nucleon density and isospin asymmetry. In particular, it has been found that the gap energy in the isospin singlet channel is strongly reduced for increasing asymmetry, and the transition to superfluidity is possible only for asymmetry values $\alpha = (n_n - n_p)/(n_n + n_p) \leq 0.35$ [26,27]. Also, the critical temperature is strongly suppressed with increasing asymmetry as it can be calculated from the gap energy as well as directly from the solution of the BCS equation.

We give some relevant expressions for a fermion system interacting via a separable potential

$$V_{\tau\tau'} = -\lambda \sum_{P,k,k'} w(k)w(k') a_{\tau}^{\dagger}(P/2+k) a_{\tau'}^{\dagger}(P/2-k) a_{\tau'}(P/2-k') a_{\tau}(P/2+k'). \quad (20)$$

We use a Yamaguchi type of potential [31] with $w(k) = (k^2 + \kappa^2)^{-1}$, and $\kappa = 1.4488 \text{ fm}^{-1}$. The interaction strength in the 1S_0 channel is only about 70 percent of the strength in the 3S_1 channel which in its original form [31] is chosen to reproduce the deuteron binding energy as well as the low-energy behavior of the free scattering phase shifts. We perform an exploratory calculation and consider the interaction strength as a parameter that will be adjusted below. Furthermore, only zero angular momentum is considered. Separable representations of more realistic interactions can be found in the literature [32].

The interaction is treated in mean-field (Hartree-Fock-Bogoliubov) approximation, allowing for an isospin-singlet pair amplitude at zero total momentum P . Diagonalizing $H^{\text{MF}} - \mu_p N_p - \mu_n N_n$ using the Bogoliubov transformation, we obtain the gap equation

$$\Delta(k) = \lambda w(k) \sum_{k'} w(k') \frac{\Delta(k')}{\sqrt{(\xi_p(k') + \xi_n(k'))^2 + 4\Delta^2(k')}} [1 - f(E_{k'}^+) - f(E_{k'}^-)] \quad (21)$$

with

$$E_k^{\pm} = \frac{1}{2} \left[\sqrt{(\xi_p(k) + \xi_n(k))^2 + 4\Delta^2(k)} \pm (\xi_p(k) - \xi_n(k)) \right] \quad (22)$$

and $\xi_p(k) = \epsilon_p(k) - \mu_p$, $\xi_n(k) = \epsilon_n(k) - \mu_n$, $f(E) = (e^{E/T} + 1)^{-1}$. $\epsilon_{\tau}(k)$ are the single-particle energies including the shift due to a mean field as given above (17). From the self-consistent solution of the gap equation, besides the trivial solution $\Delta(k) = 0$ also a solution $\Delta(k) = g w(k)$ with a finite value of g may occur.

The shift in the energy density due to the formation of a gap (condensation energy density) follows as [33]

$$\begin{aligned} \Delta\mathcal{E}(n_p, n_n) &= [\mathcal{E}_{\text{pair}}(n_p, n_n) - \mathcal{E}_{\text{norm}}(n_p, n_n)] = \\ &= \frac{2}{V} \sum_k \left\{ \frac{1}{2} \left[1 - \frac{\xi_p(k) + \xi_n(k)}{\sqrt{(\xi_p(k) + \xi_n(k))^2 + 4\Delta^2(k)}} \right] (\epsilon_p(k) + \epsilon_n(k)) \right. \\ &\quad \left. - \frac{\Delta^2(k)}{\sqrt{(\xi_p(k) + \xi_n(k))^2 + 4\Delta^2(k)}} [1 + f(E_k^+) + f(E_k^-)] \right. \\ &\quad \left. + \frac{1}{2} f(E_k^+) \left[(\epsilon_p(k) - \epsilon_n(k)) + (\epsilon_p(k) + \epsilon_n(k)) \frac{\xi_p(k) + \xi_n(k)}{\sqrt{(\xi_p(k) + \xi_n(k))^2 + 4\Delta^2(k)}} \right] \right\} \end{aligned}$$

$$\begin{aligned}
& +4 \frac{\Delta^2(k)}{\sqrt{(\xi_p(k) + \xi_n(k))^2 + 4\Delta^2(k)}} \Bigg] \\
& + \frac{1}{2} f(E_k^-) \left[(\epsilon_n(k) - \epsilon_p(k)) + (\epsilon_p(k) + \epsilon_n(k)) \frac{\xi_p(k) + \xi_n(k)}{\sqrt{(\xi_p(k) + \xi_n(k))^2 + 4\Delta^2(k)}} \right. \\
& \quad \left. +4 \frac{\Delta^2(k)}{\sqrt{(\xi_p(k) + \xi_n(k))^2 + 4\Delta^2(k)}} \right] \\
& \left. - \epsilon_p(k) f(\epsilon_p(k) - \mu_p) - \epsilon_n(k) f(\epsilon_n(k) - \mu_n) \right\}. \tag{23}
\end{aligned}$$

The chemical potentials are given by the normalization to the densities of the corresponding nucleons

$$\frac{2}{V} \sum_k f(\epsilon_\tau(k) - \mu_\tau) = n_\tau. \tag{24}$$

As usual, it is assumed that the normalization condition in the paired state gives no essential change in the corresponding chemical potentials [33].

C. Finite nuclei density profiles

For infinite nuclear matter, the energy density is calculated for homogeneous densities n_τ . In finite nuclei, the densities $n_\tau(r)$ are depending on position r . Then also the quantities considered above which are functions of the densities now are parametrically depending on r .

If the density distribution of protons and neutrons is known, the gain of the binding energy due to np pairing (condensation energy) can be estimated in LDA by the integral

$$B_{np} \approx 4\pi \int_0^\infty r^2 \Delta \mathcal{E}(n_p, n_n; r) dr. \tag{25}$$

For nuclei with N neutrons and Z protons and $A = N + Z$, we have to determine the density profiles of protons and neutrons. Exploratory calculations will be performed taking the nucleon densities from a simple potential model, normalized to the corresponding numbers of protons or neutrons.

The nucleons feel a mean field phenomenologically defined as

$$V_p(r) = V_p^{\text{nucl}}(r) + V^{\text{coul}}(r), \quad V_n(r) = V_n^{\text{nucl}}(r). \tag{26}$$

We adopt the Shlomo parameterization for the mean field [34] :

$$V_\tau^{\text{nucl}} = \frac{V_\tau^0}{1 + \exp((r - R)/d)}, \quad V_\tau^0 = -V^0 + \tau V^{\text{sym}}(N - Z)/A, \tag{27}$$

with the parameter values $V^0 = 54$ MeV, $V^{\text{sym}} = 33$ MeV, and $d = 0.7$ fm. The radius of the nuclear potential is given by the implicit equation

$$R = \frac{1.12A^{1/3} + 1.0}{[1 + (\pi d/R)^2]^{1/3}}. \tag{28}$$

For the Coulomb potential we use the charged sphere formula

$$V^{\text{coul}} = \frac{Ze^2}{2R_c} \left[3 - \frac{r^2}{R_c^2} \right] \Theta(R_c - r) + \frac{Ze^2}{r} \Theta(r - R_c), \tag{29}$$

where $e^2 = 1.44$ MeV fm, $\Theta(x)$ denotes the step function. The Coulomb radius is given by

$$R_c^2 = \frac{5}{3} \langle r^2 \rangle = C^2 \frac{1 + 10/3(\pi z/C)^2 + 7/3(\pi z/C)^4}{1 + (\pi z/C)^2}, \tag{30}$$

$$C = \frac{1.12A^{1/3}}{[1 + (\pi z/C)^2]^{1/3}}, \quad (31)$$

where $\langle r^2 \rangle$ is m.s. radius from the nuclear charge density, $z = 0.54$ fm.

Within the Thomas-Fermi approximation, at zero temperature the local density is given by

$$\begin{aligned} n_\tau(r) &= \frac{1}{3\pi^2} (k_\tau^F)^3 \Theta(\lambda_\tau - V_\tau(r)) \\ k_\tau^F &= \left[\frac{2m}{\hbar^2} (\lambda_\tau - V_\tau(r)) \right]^{1/2}, \end{aligned} \quad (32)$$

where $V_\tau(r)$ is the total potential (26). The chemical potentials λ_τ are determined by the constraints

$$Z = \int d^3r n_p(r), \quad N = \int d^3r n_n(r). \quad (33)$$

Obviously this simple treatment gives only a first estimate of the nucleonic densities. They are correctly normalized and account also for the Coulomb repulsion, but can be improved by considering the quasiparticle dispersion relation or shell effects in the nuclear density.

IV. NUMERICAL RESULTS

A. Condensation energy

In the LDA, we proceed in the following way. First, we calculate the density profiles $n_\tau(r)$ for protons and neutrons (see Sec. III C). As typical examples the density profiles for ^{40}Ar and ^{40}Ti are shown in Fig. 11. Then, with the total local density and the asymmetry $[n_n(r) - n_p(r)]/[n_n(r) + n_p(r)]$ as inputs the local gap function $\Delta(k; r)$ is obtained from the solution of the gap equation (21). This is performed within a self-consistent Hartree-Fock scheme, which gives the local single-particle energies $\epsilon_\tau(k; r)$ depending on the isospin variable. Having the pairing gap at our disposal the local condensation energy density (23) is determined. Finally, integration over the whole nucleus (25) gives the contribution to the binding energy due to np pairing.

The coupling strength λ of the separable interaction is considered as a parameter which should be taken as a phenomenological quantity. To reproduce the average Wigner energy in the mass number region $20 \leq A \leq 100$ (2.62 MeV), we have taken the value $\lambda = 92.35 \text{ MeV fm}^3$ in the $^1\text{S}_0$ channel and $\lambda = 131.50 \text{ MeV fm}^3$ in the $^3\text{S}_1$ channel. Usually the deuteron binding energy is used to adjust this parameter. However, within our simple model we should take into consideration that the interaction (20) is an effective description, simplifying different contributions as the different channels, short distance repulsion, spin-orbit coupling etc. Furthermore, some suppression of $T = 0$ pairing in symmetric nuclear matter due to medium polarization is expected. The fact that the np pair is bound (deuteron) whereas nn and pp pairs are not is essentially due to the tensor force leading to the d -wave component in the deuteron. Without this component the pn interaction in the $T = 0$ channel would hardly be different from the nn or pp interactions. The fate of the tensor force in the nuclear medium is, however, a much debated subject in nuclear physics and it is quite possible that the tensor force is much more screened than the other parts of the nuclear force [35]. In this sense the use of a bare interaction in the pn ($T = 0$) channel may be more questionable than it is in the $T = 1$ channel.

The isospin singlet gap for symmetric nuclear matter is shown in Fig. 12 for different temperatures. Below normal nuclear matter density, which is of relevance here, the difference between $T = 0$ and $T = 0.5$ MeV temperature is small. The dependence of the isospin singlet gap on the nuclear matter density is shown in Fig. 13 for the temperature $T = 0.75$ MeV and different asymmetries. The gap is strongly reduced for increasing asymmetries and temperatures confirming the results obtained previously in [26,27].

For a nucleus with $A = 40$, the average gap on the Fermi level [17]

$$\Delta = \sum_\tau \int d^3r \Delta(k_F(r); r) [n_\tau(r)]^{1/3} / \sum_\tau \int d^3r [n_\tau(r)]^{1/3} \quad (34)$$

is shown as function of the asymmetry in Fig. 14 for different temperatures.

The condensation energy as a function of asymmetry is shown in Fig. 15 for different temperatures. For the calculation, a fixed $A = 40$ was assumed. Below $T = 0.5$ MeV, the dependence on temperature is negligible (see

also Fig. 12), but becomes strong for $T > 1$ MeV. The influence of the Coulomb interaction taken into account for the calculation of the density profiles destroys the symmetry with respect to $N - Z = 0$. The Coulomb effect is to increase the overlap between neutron and proton densities in the tail of the density profiles, as shown in Fig. 11. As a consequence the pairing gap is slightly enhanced. Furthermore, the steep decrease of the condensation energy near $N - Z = 5$ is also shown. This is in correspondence to the findings given in Fig. 4.

B. Quartetting

The additional contribution due to quartetting seems to be high in the region of light nuclei, where the α cluster model is a good approximation. The strong even-odd staggering is reduced at higher masses.

To give an estimation of the effect of quartetting, the evaluation of the np condensation energy (25) was repeated for symmetric nuclei ($Z = N$), where the magnitude of the gap $\Delta(k; r)$ was increased in the density region where quartetting can occur, i.e. at densities below 0.03 fm^{-3} . In detail, the gap was increased by a factor, which was obtained from the ratio T_4^c/T_t^c of the corresponding critical temperatures at given density n , as shown in Fig. 10.

Comparing with calculations neglecting quartetting, the gain of binding energy due to quartetting has been evaluated for different nuclei with $Z = N$. Exploratory calculations for nuclei of medium size ($A \approx 100$) show that the contribution due to quartetting is almost zero but may become large for small A . For instance, the calculation for ^{12}C and ^{16}O give an additional contribution to the condensation energy due to quartetting of 10.3 and 9.6 percent, respectively, if compared with isospin singlet pairing.

C. Excited Nuclei

After discussing the contribution of pn -pairing to the nucleon binding energy, comparing with the pairing energy in asymmetric nuclear matter, it is of interest to discuss also the effect of excitations. In nuclear matter, excitations are well understood in the context of finite temperatures. It is expected that the effects of condensates are decreasing with increasing excitation.

To investigate the effect of excitations on the formation of condensates in finite nuclei, we analyse the 2^+ excitations of even-even nuclei. In particular, we used the w -filter for the analysis and compared $w_{2^+}(Z, Z)$ for the excited nuclei with $w_0(Z, Z)$ for the ground state nuclei, see Fig. 16. We determined the mean value of the ratio from $Z = 10$ to $Z = 26$ and obtained $w_{2^+}(Z, Z)/w_0(Z, Z) = 0.64 \pm 0.19$.

This result can be compared with the influence of finite temperature on the pairing in nuclear matter. The average 2^+ excitation energy of even-even nuclei, taken for the interval $10 \leq Z \leq 30$, is 1750 keV. Performing a finite temperature Thomas-Fermi calculation for a nucleus of medium proton number $Z = 20$, this excitation energy would correspond to a temperature of about 1 MeV.

As shown in Fig. 15, similar to the decrease of the gap we observe also a decrease of the value $b_0(Z)$ with increasing temperature. The result of the calculation is in agreement with the empirical value given above.

V. CONCLUSIONS

In this work we have shown that isospin-singlet pairing and α like quartetting may contribute to the binding energy of nuclei very close to the symmetry line $N = Z$. These contributions are relatively large for smaller nuclei ($Z < 20$). For medium mass nuclei neutron-proton pairing in the isoscalar channel disappears already for $|N - Z| = 4$. This stems from the very rapid decrease of the isoscalar pairing as a function of the unbalance in the Fermi energies of protons and neutrons. These facts can explain the origin of the Wigner term in the mass formula as well as the empirically determined $\Delta B(Z, N)$ or $b_{Z-N}(Z)$, respectively (see Eqs. (1), (10)). An enhancement of the isoscalar pairing contribution to the binding energy is obtained if in addition quartetting is taken into account. An interesting effect is that the reduction of the condensation energy with increasing excitation of the nuclei seems to be in agreement with empirical data.

Our calculations are exploratory in the sense that they were performed in the rather crude LDA approach. However, please notice that the LDA has yielded in the past quite reasonable results on the average, i.e. for a gap averaged over the shell effects [17]. Therefore, we think that our results give a quite reliable first orientation of the effect. The approach should be improved in several respects. First a more realistic force should be employed. Second shell effects must properly be included. Eventually, effects of number projection, even-odd staggering and pair fluctuations should also be investigated. Such studies shall be performed in the future. It is the hope that isoscalar pairing and

quartetting will give us precious hints on the effective neutron-proton interaction in a nuclear medium as well as very interesting clustering and condensation phenomena in nuclei.

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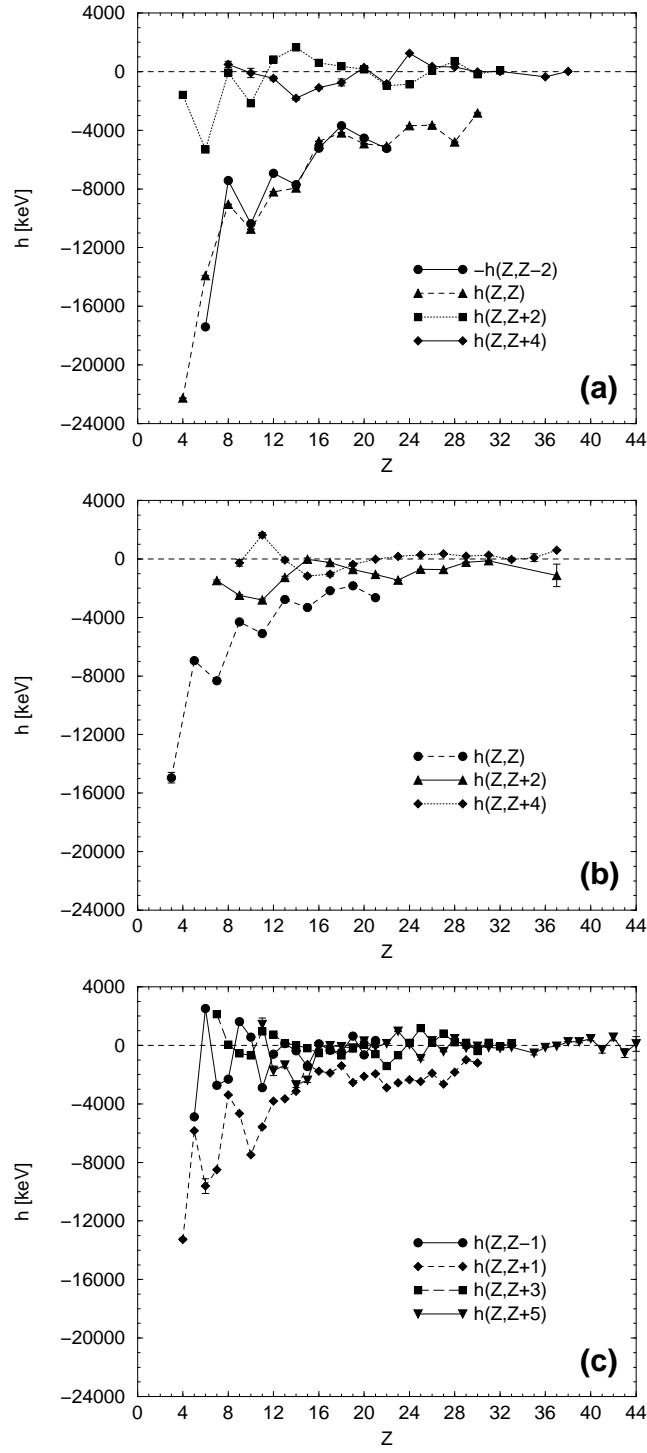


FIG. 1. The filter $h(Z, Z+i)$, Eq. (3), is given for various i as it can be extracted from the experimental binding energies. Results are separately shown for even-even (a), odd-odd (b), and even-odd/odd-even (c) nuclei.

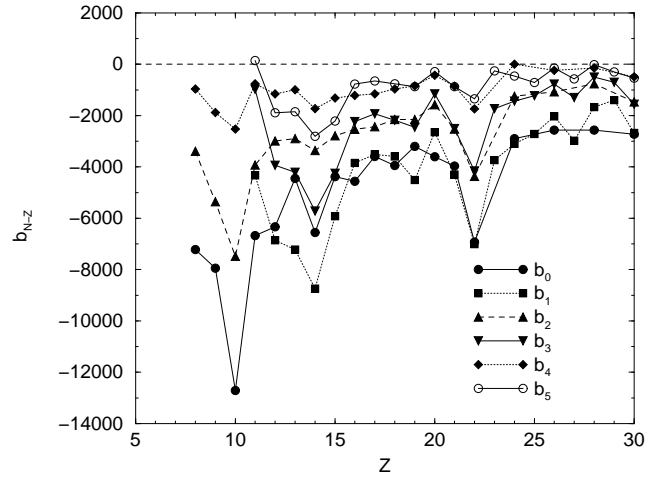


FIG. 2. The parameter $b_{N-Z}(Z)$ for even and odd proton number Z as it is derived from the filter h .

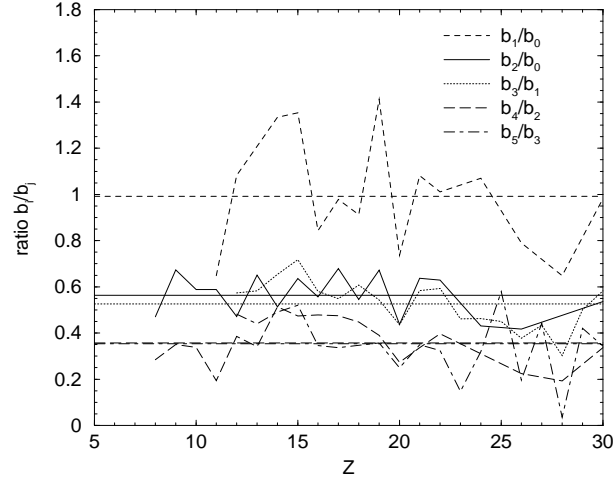


FIG. 3. A selection of several ratios b_i/b_j as a function of Z . The dashed lines represent the average values of the ratios.

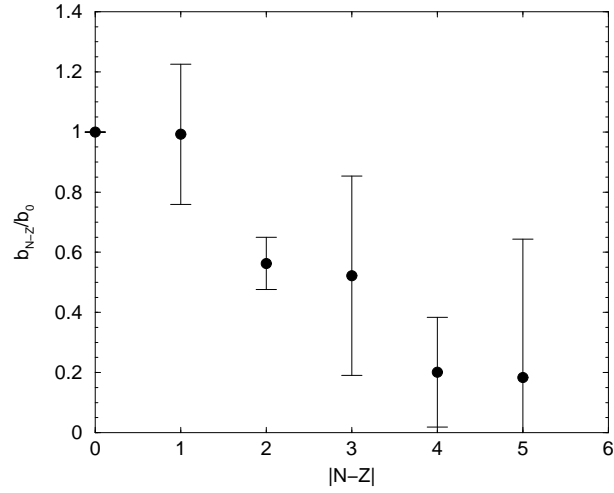


FIG. 4. Averages of the ratio b_{N-Z}/b_0 as a function of the absolute value of the difference between protons and neutrons.

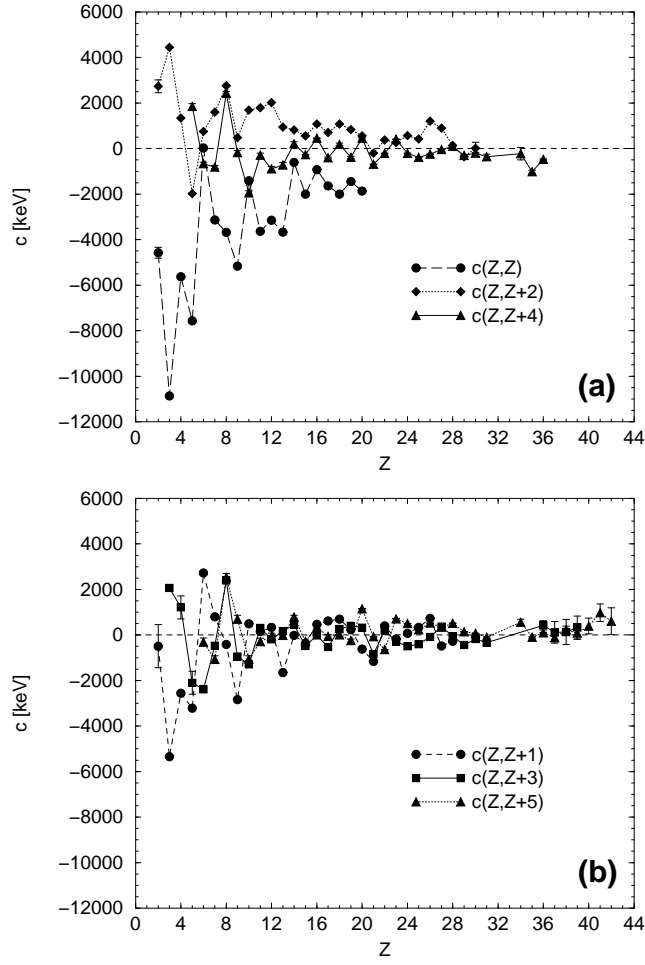


FIG. 5. The filter $c(Z, Z+i)$ for various i . Results are separately shown for even-even/odd-odd (a) and even-odd/odd-even (b) nuclei.

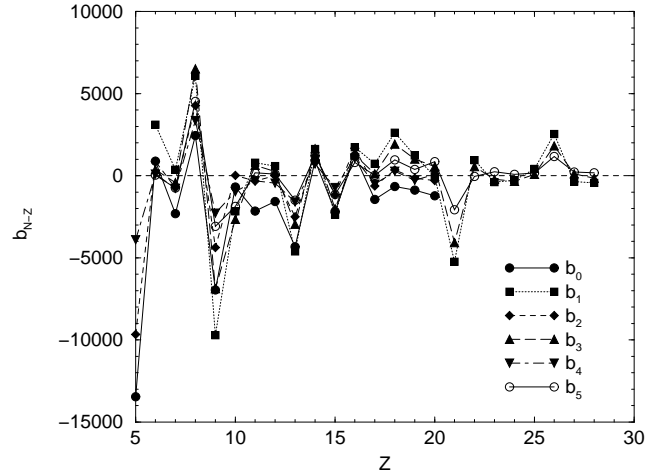


FIG. 6. The parameter $\delta b_{N-Z}(Z)$ as derived from the filter c , Eq. (9), for even and odd proton number Z .

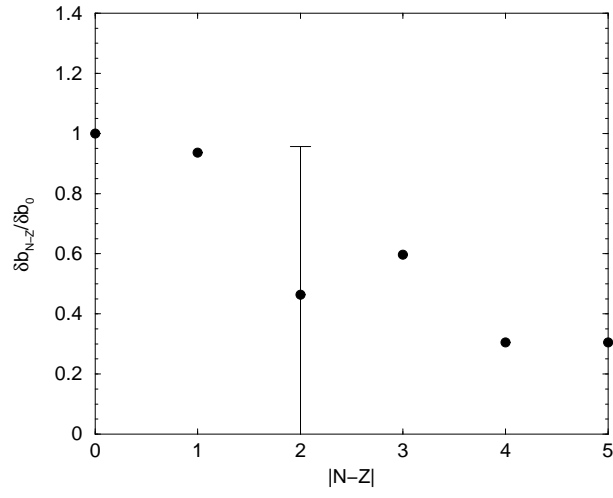


FIG. 7. Averages of the ratio $\delta b_{N-Z} / \delta b_0$ as a function of the absolute value of the proton-neutron difference.

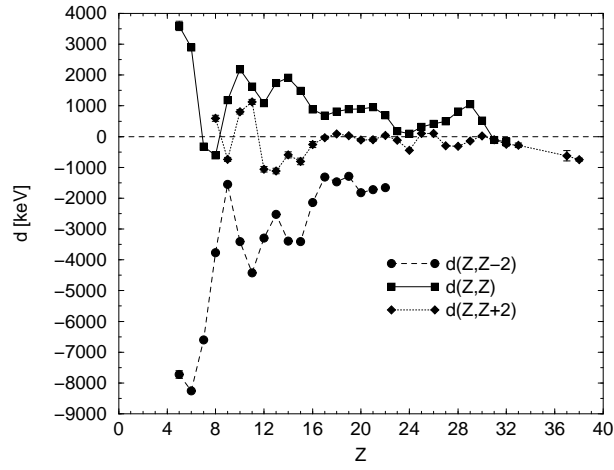


FIG. 8. For demonstration purposes the filter $d(Z, Z+i)$ for various i is given considering only even-even and odd-odd nuclei.

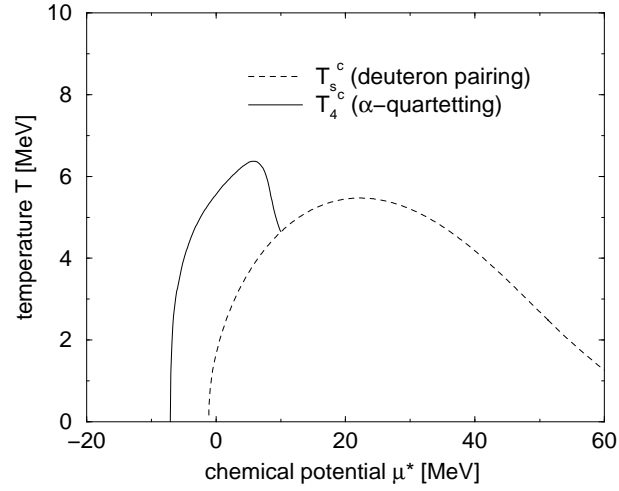


FIG. 9. Critical temperatures for the onset of quantum condensation in symmetric nuclear matter, model calculation. The critical temperature of the onset of two-particle pairing T_t^c is compared with T_4^c for the onset of a four-particle condensate, as a function of the chemical potential.

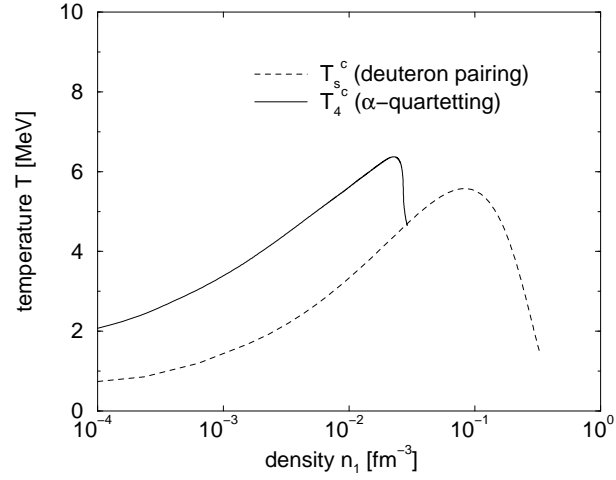


FIG. 10. The same as Fig. 9 but as a function of the uncorrelated density.

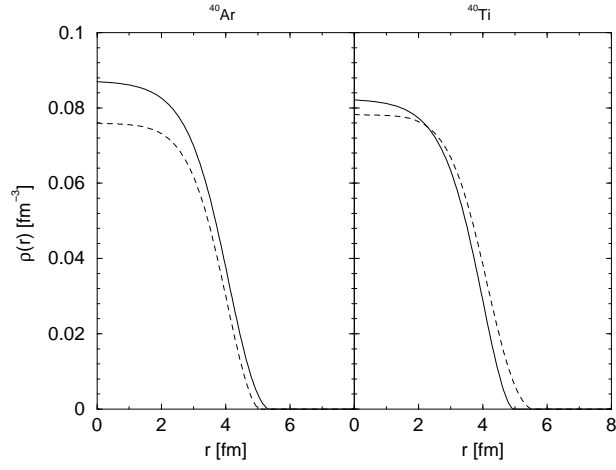


FIG. 11. Density profiles of neutrons (solid lines) and protons (dashed lines) calculated within the Thomas-Fermi approximation for ^{40}Ar and ^{40}Ti at zero temperature.

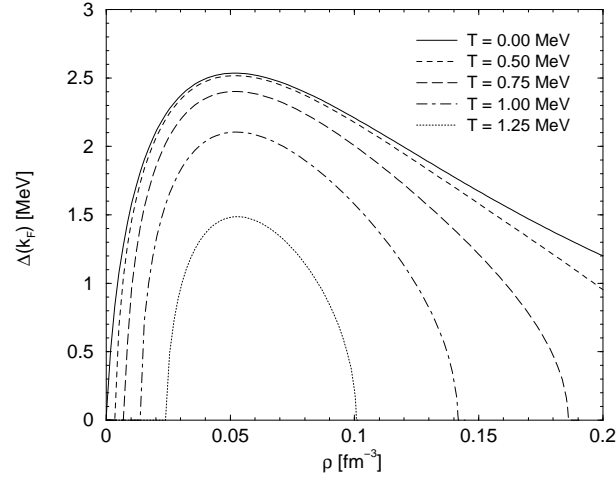


FIG. 12. Proton-neutron pairing gap for asymmetric nuclear matter as a function of the total density for symmetric nuclear matter and different temperatures T .

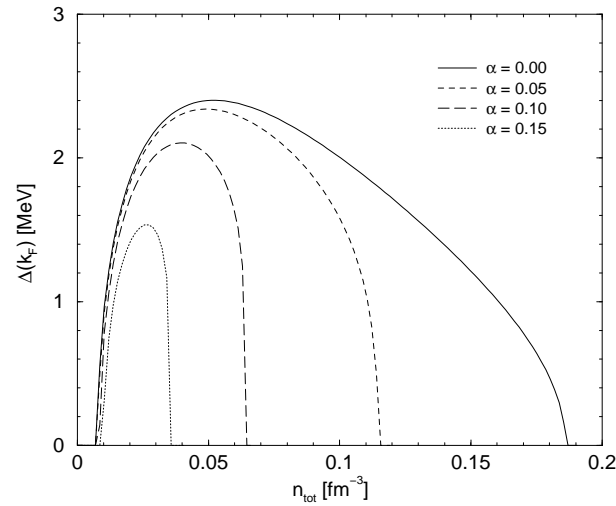


FIG. 13. Proton-neutron pairing gap for asymmetric nuclear matter as a function of the total density at $T = 0.75$ MeV temperature and four values of the asymmetry parameter α .

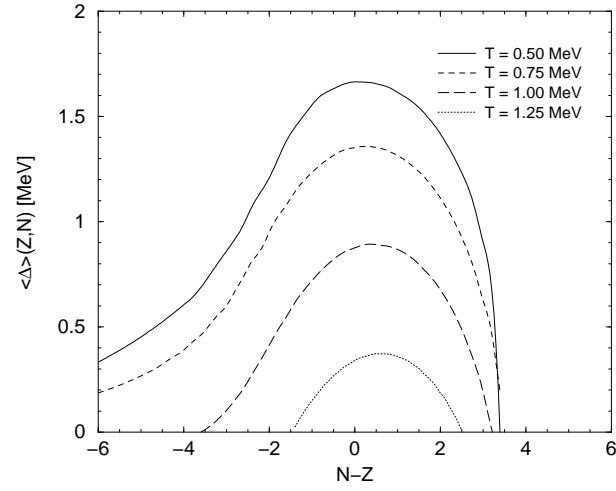


FIG. 14. Average proton-neutron pairing gap of nuclei with a mass number $A = 40$ as a function of the asymmetry $N - Z$ and three values of the temperature.

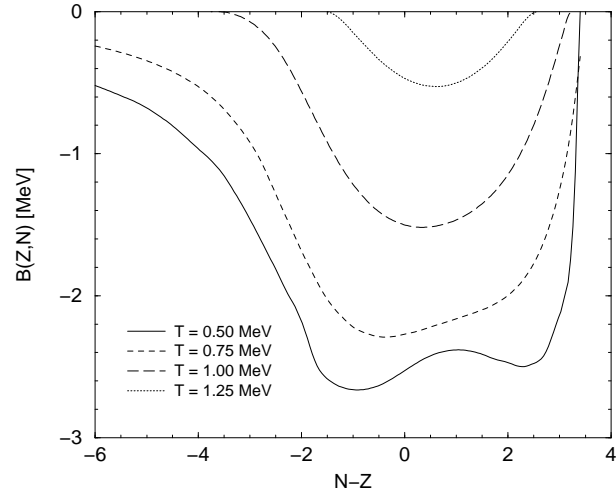


FIG. 15. Condensation energy of nuclei with a mass number $A = 40$ as a function of the asymmetry $N - Z$ and three values of the temperature.

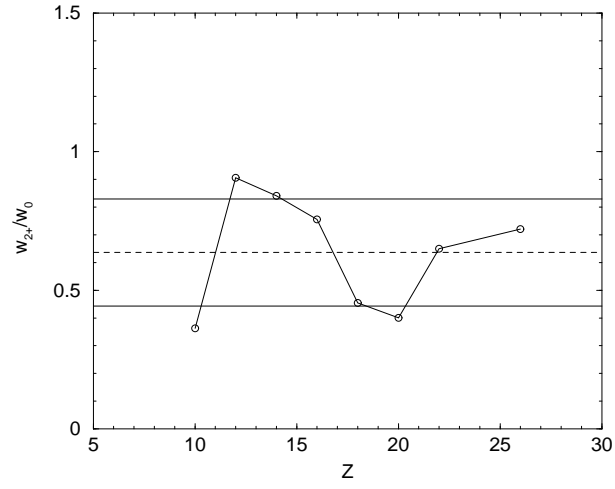


FIG. 16. For the filter $w(Z, Z)$ the ratio w_{2+}/w_0 of 2^+ excited nuclei over ground state nuclei is given for even proton and neutron number. The average value is indicated by the dashed lines as well as the statistical error bars by the thin lines.